

# Space-like and time-like pion-rho transition form factors in the light-cone formalism

Jianghao Yu<sup>1</sup>, Bo-Wen Xiao<sup>2</sup> and Bo-Qiang Ma<sup>1,3</sup>

<sup>1</sup>Department of Physics, Peking University, Beijing 100871, China

<sup>2</sup>Department of Physics, Columbia University, New York, NY 10027, USA

<sup>3</sup>MOE Key Laboratory of Heavy Ion Physics, Peking University, Beijing 100871, China

E-mail: mabq@th.phy.pku.edu.cn

## Abstract.

Having calculated the light-cone wave function of the pseudoscalar meson by using two equivalent and fully covariant methods, we generalize such methods to the valence Fock states of the vector meson in the light-cone formalism. We investigate the decay constant of the  $\rho$  meson  $f_\rho$ , the  $\gamma^*\pi \rightarrow \rho$  and  $\gamma^*\rho \rightarrow \pi$  transition form factors and especially the transition magnetic moments. By using two groups of constraint parameters, we predict the space-like and time-like form factors  $F_{\pi\rho}(Q^2)$  and  $F_{\rho\pi}(Q^2)$  at low and moderate energy scale and the electromagnetic radius of these transition processes. In addition, we extend our calculation to  $\gamma^*\pi \rightarrow \omega$  space-like and time-like form factors by using the same sets of parameters.

## 1. Introduction

To investigate the electromagnetic transition processes between the pseudoscalar meson and the vector meson is an important and distinctive way to understand the internal structure of hadrons. It is well known that, at the low and moderately high four momentum transfer  $Q^2$ , the elastic and transition form factors of exclusive processes have to be treated non-perturbatively due to the large coupling constant of QCD and bound state effects. In non-perturbative region, the intrinsic momentum of quarks in a meson has the same order of magnitude as that of the quark mass. Therefore, the consistent treatment of relativistic effects of the quark motion and spin in a bound state becomes a main issue. The light-cone (LC) formalism [1, 2] provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom. Its application to exclusive processes has mainly been developed in the light-cone Fock representation [2, 3, 4, 5].

Conventionally, the exclusive form factor can be calculated in terms of the overlap of light-cone wave functions in the Drell-Yan frame [6], which refer to the valence contribution. However, in many cases the non-valence section will contribute to the form factor, even in the  $q^+ = 0$  frame, which refer to the zero-mode contribution [7, 8, 9]. Therefore, we need to treat the form factor in the light-cone formalism carefully. In this paper, we derived the spin-flavor section of valence light-cone  $\rho$  meson wave function through the Melosh transformation [10] of the instant SU(6) quark model. On the other hand, from one kind of the covariant  $\rho$ -meson-quark-antiquark vertex we also obtain this same spin-flavor section of the light-cone  $\rho$  meson wave function. In Ref. [9, 11, 8], this vertex has been used to light-cone quark model for exclusive processes. Bakker, Choi and Ji [8] used this vector meson vertex to calculate the transition form factors between pseudoscalar and vector mesons in light-cone dynamics. They concluded that the zero mode contribution vanished in the  $q^+ = 0$  frame. Therefore, it is fortunate that we can discuss the valence contribution only. For the momentum space section of light-cone wave function, Bakker *et al.* used the perturbative energy denominator to analyse the non-valence structure of the wave function. But apparently hadronic wave function are non-perturbative, so a broadly used Brodsky-Huang-Lepage wave function [12] can reflect some feature of the non-perturbative effect. In this paper, we specify the pseudoscalar and vector meson to pion-rho transition process. Several papers have discussed for this process. Choi and Ji [13] used a different covariant  $\rho$ -quark-antiquark vertex [14] to discuss the space-like  $\rho \rightarrow \pi\gamma^*$  process. Cardarelli *et al.* [15] summed the Dirac and Pauli form factors in the space-like  $\gamma^*\pi^+ \rightarrow \rho^+$  process, but Pauli form factor perhaps has non-vanishing zero mode contribution and the paper did not verify this. We will give an unified treatment for the  $\gamma^*\pi \rightarrow \rho$  process in the space-like and time-like region and give a more careful description about  $\rho$  meson decay properties, transition magnetic moments and the electromagnetic radius. Furthermore, other approaches have been adopted in order to understand these transition form factors, such as the lattice technique by Edwards [16] for the  $\gamma^*\rho \rightarrow \pi$  transition, the light-cone QCD sum rule calculation by Khodjamirian [17] for the  $\gamma^*\rho \rightarrow \pi$  transition, and the model based on Dyson-Schwinger equation [18].

The paper is organized as follow: we derive the wavefunction of the vector meson in Section 2. In Section 3 and Section 4, we compute the transition form factors and decay widths, respectively. Finally, we provide numerical analysis results of theoretical calculations. We present numerical analysis and conclusion in Section 5 and 6.

## 2. The valence Fock state of the rho meson

The light-front Fock expansion of any hadronic system is constructed by quantizing QCD at fixed light-cone time  $x^+ = x^0 + x^3$  and by establishing the invariant light-cone Hamiltonian  $H_{LC}$ :  $H_{LC} = P^+P^- - \vec{P}_\perp^2$  [1, 2]. In principle, solving the  $H_{LC}$  eigenvalue problem gives rise to the entire mass spectrum of color-singlet hadron states in QCD, as well as their corresponding light-front wave functions. In particular, the hadronic state satisfies  $H_{LC}|\psi_H\rangle = M^2|\psi_H\rangle$ , where  $|\psi_H\rangle$  is an expansion in multi-particle Fock states. Considering a meson with momentum  $P$  and spin projection  $S_z$ , one can expand the hadronic eigenstate  $|\psi_M\rangle$  in QCD in terms of eigenstates  $\{|n\rangle\}$ :

$$\begin{aligned} |\psi_M(P^+, \vec{P}_\perp, S_z)\rangle = \sum_{n, \lambda_i \in n} \int \prod_{i=1}^n \left( \frac{dx_i d^2\vec{k}_{\perp i}}{2\sqrt{x_i} (2\pi)^3} \right) 16\pi^3 \delta\left(1 - \sum_{i=1}^n x_i\right) \delta^{(2)}\left(\sum_{i=1}^n \vec{k}_{\perp i}\right) \\ \times \psi_{n/M}^{S_z}(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle, \end{aligned} \quad (1)$$

where the Fock state  $n$  contains  $n$  constituents and  $\lambda_i$  is the helicity of the  $i$ -th constituent. Here  $x_i = k_i^+/P^+$  are boost-invariant light-cone longitudinal momentum fractions and  $\vec{k}_{\perp i}$  represent the transverse momentum of the  $i$ -th constituent in Fock state  $n$  in the center of mass frame. The  $n$ -particle Fock states are normalized as follows:

$$\langle n; p_i'^+, \vec{p}'_{\perp i}, \lambda_i' | n; p_i^+, \vec{p}_{\perp i}, \lambda_i \rangle = \prod_{i=1}^n \left( 16\pi^3 p_i^+ \delta(p_i'^+ - p_i^+) \delta^{(2)}(\vec{p}'_{\perp i} - \vec{p}_{\perp i}) \delta_{\lambda_i' \lambda_i} \right) \quad (2)$$

To simplify the problem, we construct the light-cone wavefunctions of mesons according to the valence quark model. Since the high Fock states of hadron are suppressed [4], we only take the valence states of the light-cone wave function into account, which corresponds to the minimal Fock states or the first order contributions in the full calculation.

For the pion meson, we have derived the valence light-cone wavefunctions by employing two different methods [19]. In the following, we summarize the main idea of two methods. On one hand, one can write the vertex of the pseudoscalar (e.g., pion) with two spin- $\frac{1}{2}$  fermions (e.g., quark and antiquark) inside as the following:

$$\frac{\bar{u}(p_1^+, p_1^-, \vec{k}_\perp)}{\sqrt{p_1^+}} \gamma_5 \frac{v(p_2^+, p_2^-, -\vec{k}_\perp)}{\sqrt{p_2^+}}, \quad (3)$$

where  $\bar{u}(p_1^+, p_1^-, \vec{k}_\perp)$  and  $v(p_2^+, p_2^-, -\vec{k}_\perp)$  are the light-cone spinors of the quark and the antiquark, respectively. In this full relativistic field theory treatment, this matrix element of the interaction vertex (3) can be taken as the minimal Fock wavefunctions of pseudoscalars. On the other hand, the same problem can be analyzed in the light-cone quark model. In this model, the light-cone wave function of a composite system can be obtained by transforming the ordinary equal-time (instant-form) wave function in the rest frame into that in the light-front dynamics, by taking into account the relativistic effects such as the Melosh-Wigner rotation effect [20, 21]. The transformation for the spin space wave functions between the two formalisms is accomplished by the use of the Melosh-Wigner rotation. The Melosh-Wigner rotation is one of the most important ingredients of the light-cone formalism, and it relates the light-cone spin

state  $|J, \lambda\rangle_F$  to the ordinary instant-form spin state wave functions  $|J, s\rangle_T$  by the general relation [10, 22]

$$|J, \lambda\rangle_F = \sum_s U_{s\lambda}^J |J, s\rangle_T. \quad (4)$$

The transformation for the momentum space wave functions becomes possible with the help of some Ansatz such as the Brodsky-Huang-Lepage prescription [12]. The equivalence of the two approaches has been demonstrated in Ref. [19], where both approaches are used to derive the pion light-cone wave function and lead to the same result.

Hereafter, in a similar way, we extend such idea to vector mesons and derive the light-cone wave functions of the  $\rho$  meson as an example. In the end, we also provide similar calculation respect to the vector meson  $\omega$ .

### 2.1. Vector meson wave-functions from Melosh-Wigner rotation.

Using the Melosh-Wigner rotation, we derive the light-cone wave function for the quark-antiquark Fock state of the  $\rho$  meson in the light-cone quark model. The spin parts of the  $\rho$  wave function in the SU(6) quark-antiquark model in the instant form are written as: for the  $\rho^+$  meson

$$\begin{aligned} |\rho_{\pm}^+\rangle &= u^{\uparrow, \downarrow} \bar{d}^{\uparrow, \downarrow} \\ |\rho_0^+\rangle &= \frac{1}{\sqrt{2}} (u^{\uparrow} \bar{d}^{\downarrow} + u^{\downarrow} \bar{d}^{\uparrow}); \end{aligned} \quad (5)$$

$u \leftrightarrow d$  in (5) corresponds to the  $\rho^-$  meson. Applying the transformation in equation (4) on both sides of equation (5), we obtain the spin space wave function of the  $\rho^+$  in the light-front frame. The Melosh-Wigner transformation relates the instant-form and light-front form spin states with four-momentum  $(k^0, \vec{k})$  as follows

$$\begin{aligned} \chi_q^{\uparrow}(T) &= w_q [(k^+ + m) \chi_q^{\uparrow}(F) - k^R \chi_q^{\downarrow}(F)], \\ \chi_q^{\downarrow}(T) &= w_q [(k^+ + m) \chi_q^{\downarrow}(F) + k^L \chi_q^{\uparrow}(F)], \end{aligned} \quad (6)$$

where  $w_q = [2k^+(k^0 + m)]^{-\frac{1}{2}}$ ,  $k^{R,L} = k^1 \pm ik^2$ , and  $k^+ = k^0 + k^3 = x\mathcal{M}$ ,  $m$  is the mass of the quark, and  $\mathcal{M}^2 = \frac{\vec{k}_{\perp}^2 + m^2}{x(1-x)}$  is the invariance mass of the composite state. Therefore the spin part of the light-cone wave function of the  $\rho^+$  reads

$$\psi_{\rho^+}^{S_z}(x, \vec{k}_{\perp}) = \sum_{\lambda_1, \lambda_2} C_{S_z}^F(x, \vec{k}_{\perp}, \lambda_1, \lambda_2) \chi_1^{\lambda_1}(F) \chi_2^{\lambda_2}(F), \quad (7)$$

where  $S_z$  and  $\lambda$  are the spin projection of the  $\rho^+$  meson and the helicity of the quark. For the rho meson  $\rho_+^+$  with  $S_z = +1$ , the coefficients of the spin wave function are

$$\begin{aligned} C_{+1}^F(x, \vec{k}_{\perp}, \uparrow, \uparrow) &= \omega^{-1} (m(\mathcal{M} + 2m) + \vec{k}_{\perp}^2); \\ C_{+1}^F(x, \vec{k}_{\perp}, \uparrow, \downarrow) &= \omega^{-1} (x\mathcal{M} + m) k^R; \\ C_{+1}^F(x, \vec{k}_{\perp}, \downarrow, \uparrow) &= -\omega^{-1} ((1-x)\mathcal{M} + m) k^R; \\ C_{+1}^F(x, \vec{k}_{\perp}, \downarrow, \downarrow) &= -\omega^{-1} (k^R)^2, \end{aligned} \quad (8)$$

where  $\omega = (\mathcal{M} + 2m) \sqrt{\vec{k}_{\perp}^2 + m^2}$ . The coefficients of the  $\rho_0^+$  are

$$C_0^F(x, \vec{k}_{\perp}, \uparrow, \uparrow) = \omega^{-1} ((1-x)\mathcal{M} - x\mathcal{M}) k^L / \sqrt{2};$$

$$\begin{aligned}
C_0^F(x, \vec{k}_\perp, \uparrow, \downarrow) &= \omega^{-1}(m(\mathcal{M} + 2m) + 2\vec{k}_\perp^2)/\sqrt{2}; \\
C_0^F(x, \vec{k}_\perp, \downarrow, \uparrow) &= \omega^{-1}(m(\mathcal{M} + 2m) + 2\vec{k}_\perp^2)/\sqrt{2}; \\
C_0^F(x, \vec{k}_\perp, \downarrow, \downarrow) &= \omega^{-1}(x\mathcal{M} - (1-x)\mathcal{M})k^R/\sqrt{2}.
\end{aligned} \tag{9}$$

And the coefficients of the  $\rho^\pm$  are

$$\begin{aligned}
C_{-1}^F(x, \vec{k}_\perp, \uparrow, \uparrow) &= -\omega^{-1}(k^L)^2; \\
C_{-1}^F(x, \vec{k}_\perp, \uparrow, \downarrow) &= \omega^{-1}((1-x)\mathcal{M} + m)k^L; \\
C_{-1}^F(x, \vec{k}_\perp, \downarrow, \uparrow) &= -\omega^{-1}(x\mathcal{M} + m)k^L; \\
C_{-1}^F(x, \vec{k}_\perp, \downarrow, \downarrow) &= \omega^{-1}(m(\mathcal{M} + 2m) + \vec{k}_\perp^2).
\end{aligned} \tag{10}$$

All of the component coefficients  $C_{S_z}^F(x, \vec{k}_\perp, \lambda_1, \lambda_2)$  satisfy the unitary relation,

$$\sum_{\lambda_1, \lambda_2} C_{S_z}^F(x, \vec{k}_\perp, \lambda_1, \lambda_2)^* C_{S_z}^F(x, \vec{k}_\perp, \lambda_1, \lambda_2) = 1. \tag{11}$$

Therefore, the Fock state expansion of the valence wave function for the  $\rho^+$  can be expressed as

$$\begin{aligned}
\left| \psi_{\rho^+} \left( P^+, \vec{P}_\perp, S_z \right) \right\rangle &= \int \frac{d^2 \vec{k}_\perp dx}{16\pi^3} \left[ \psi_{\rho^+}^{S_z}(x, \vec{k}_\perp, \uparrow, \uparrow) \left| xP^+, \vec{k}_\perp, \uparrow, \uparrow \right\rangle \right. \\
&\quad + \psi_{\rho^+}^{S_z}(x, \vec{k}_\perp, \uparrow, \downarrow) \left| xP^+, \vec{k}_\perp, \uparrow, \downarrow \right\rangle + \psi_{\rho^+}^{S_z}(x, \vec{k}_\perp, \downarrow, \uparrow) \left| xP^+, \vec{k}_\perp, \downarrow, \uparrow \right\rangle \\
&\quad \left. + \psi_{\rho^+}^{S_z}(x, \vec{k}_\perp, \downarrow, \downarrow) \left| xP^+, \vec{k}_\perp, \downarrow, \downarrow \right\rangle \right].
\end{aligned} \tag{12}$$

Here the Fock state projection  $\psi_{\rho^+}^{S_z}$  connects to the wavefunction in the light-cone quark model as follow

$$\psi_{\rho^+}^{S_z}(x, \vec{k}_\perp, \lambda_1, \lambda_2) = C_{S_z}^F(x, \vec{k}_\perp, \lambda_1, \lambda_2) \varphi_{\rho^+}(x, \vec{k}_\perp), \tag{13}$$

where  $\varphi_{\rho^+}(x, \vec{k}_\perp)$  is the momentum wave function in the light-cone formalism. In ref.[21], Huang, Ma and Shen analyzed several valence wave function. Here we employ the Brodsky-Huang-Lepage (BHL) prescription [12] which is used in many light-cone quark models [13, 27]. This wave function is as follow

$$\varphi_{\rho^+}(x, \vec{k}_\perp) = A \exp \left[ -\frac{1}{8\beta^2} \frac{\vec{k}_\perp^2 + m^2}{x(1-x)} \right]. \tag{14}$$

From above, one can find that the Fock expansion of the two particle Fock-state for  $\rho$  meson with definite  $S_z$  has four possible spin configurations, in which each configuration have different  $\lambda_1$  and  $\lambda_2$  and satisfy the spin sum rule:  $S_z = \lambda_1 + \lambda_2 + l_z$ . While the spin of each constituent undergoes a Melosh-Wigner rotation, the spin components for these constituents does not necessarily conserve the naive spin sum  $S_z = \lambda_1 + \lambda_2$  since there are higher helicity components [20, 21, 23, 24]. Such higher helicity components plays an important role to understand the proton “spin puzzle” in the nucleon case [22, 25].

## 2.2. Vector meson wave-functions from relativistic field theory.

To obtain the spin wave function of the vector meson, we adopt another way which is based on the full relativistic field theory treatment of the interaction vertex following with the idea in [4, 5, 19]. Considering the  $\rho$  vertices connecting to the two

spin- $\frac{1}{2}$  fermions (valence quark and anti-quark), we choose the momentums in the standard light-cone intrinsic frame

$$\begin{aligned}
P &= (P^+, \frac{M^2}{P^+}, \vec{0}_\perp), \\
k_1 &= (xP^+, \frac{\vec{k}_1^2 + m^2}{xP^+}, \vec{k}_\perp), \\
k_2 &= ((1-x)P^+, \frac{\vec{k}_2^2 + m^2}{(1-x)P^+}, -\vec{k}_\perp), \\
q &= (0, \frac{Q^2}{P^+}, \vec{q}_\perp), \\
P' &= (P^+, \frac{M_\rho^2 + \vec{q}_\perp^2}{P^+}, \vec{q}_\perp),
\end{aligned} \tag{15}$$

where  $P$  and  $P'$  are the momenta of the vector meson before interaction and after interaction, respectively;  $k_1$  and  $k_2$  are the momenta of the quark and anti-quark, respectively;  $q$  is the momentum of the virtual photon. In addition, the polarization vectors of the  $\rho$  meson used in this analysis are given by

$$\begin{aligned}
\epsilon_\pm^\mu &= (\epsilon_\pm^+, \epsilon_\pm^-, \epsilon_\pm^\perp) = \left( 0, \frac{\mp\sqrt{2}P_\perp^{R,L}}{P^+}, \frac{\mp 1}{\sqrt{2}}, \frac{-i}{\sqrt{2}} \right), \\
\epsilon_0^\mu &= (\epsilon_0^+, \epsilon_0^-, \epsilon_0^\perp) = \frac{1}{M} \left( P^+, \frac{\vec{P}_\perp^2 - M^2}{P^+}, \vec{P}_\perp \right).
\end{aligned} \tag{16}$$

The matrix element of the effective vertex which had used in [8, 9, 11] is

$$\bar{u}(k_1^+, k_1^-, \vec{k}_\perp, \lambda_1) \left( \gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m} \right) \cdot \epsilon_{S_z} v(k_2^+, k_2^-, -\vec{k}_\perp, \lambda_2). \tag{17}$$

We can obtain the Fock expansion of the two-particle Fock state of the  $\rho$  meson

$$\begin{aligned}
\left| \psi_{\rho^+} \left( P^+, \vec{P}_\perp, S_z \right) \right\rangle &= \int \frac{d^2 \vec{k}_\perp dx}{16\pi^3} \varphi_{\rho^+}(x, \vec{k}_\perp) \\
&\times \bar{u}(k_1^+, k_1^-, \vec{k}_\perp, \lambda_1) \left( \gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m} \right) \cdot \epsilon_{S_z} v(k_2^+, k_2^-, -\vec{k}_\perp, \lambda_2) \\
&\left| xP^+, \vec{k}_\perp, \lambda_1, \lambda_2 \right\rangle,
\end{aligned} \tag{18}$$

where  $\varphi_{\rho^+}(x, \vec{k}_\perp)$  is the momentum space wavefunction of the  $\rho$  meson. According to the spinor representations of  $\bar{u}$  and  $v$  in the Appendix of the ref [2], we calculate the above matrix element and obtain the following results:

$$\begin{aligned}
\bar{u}_\uparrow \left( \gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m} \right) \cdot \epsilon_+ v_\uparrow &= -\frac{\sqrt{2}(m(\mathcal{M} + 2m) + \vec{k}_1^2)}{\sqrt{x(1-x)(\mathcal{M} + 2m)}}, \\
\bar{u}_\uparrow \left( \gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m} \right) \cdot \epsilon_+ v_\downarrow &= -\frac{\sqrt{2}(k_1^+ + m)k^R}{\sqrt{x(1-x)(\mathcal{M} + 2m)}}, \\
\bar{u}_\downarrow \left( \gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m} \right) \cdot \epsilon_+ v_\uparrow &= \frac{\sqrt{2}(k_2^+ + m)k^R}{\sqrt{x(1-x)(\mathcal{M} + 2m)}}, \\
\bar{u}_\downarrow \left( \gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m} \right) \cdot \epsilon_+ v_\downarrow &= \frac{\sqrt{2}(k^R)^2}{\sqrt{x(1-x)(\mathcal{M} + 2m)}},
\end{aligned} \tag{19}$$

for the  $\rho^+$  meson with spin projection  $S_z = +1$ ,

$$\begin{aligned}
\bar{u}_\uparrow \left( \gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m} \right) \cdot \epsilon_0 v_\uparrow &= -\frac{(k_2^+ - k_1^+)k^L}{\sqrt{x(1-x)(\mathcal{M} + 2m)}}, \\
\bar{u}_\uparrow \left( \gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m} \right) \cdot \epsilon_0 v_\downarrow &= -\frac{m(\mathcal{M} + 2m) + 2\vec{k}_1^2}{\sqrt{x(1-x)(\mathcal{M} + 2m)}}, \\
\bar{u}_\downarrow \left( \gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m} \right) \cdot \epsilon_0 v_\uparrow &= -\frac{m(\mathcal{M} + 2m) + 2\vec{k}_1^2}{\sqrt{x(1-x)(\mathcal{M} + 2m)}}, \\
\bar{u}_\downarrow \left( \gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m} \right) \cdot \epsilon_0 v_\downarrow &= -\frac{(k_1^+ - k_2^+)k^R}{\sqrt{x(1-x)(\mathcal{M} + 2m)}},
\end{aligned} \tag{20}$$

for the  $\rho^+$  meson with spin projection  $S_z = 0$ , and

$$\begin{aligned}
\bar{u}_\uparrow(\gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m}) \cdot \epsilon_- v_\uparrow &= \frac{\sqrt{2}(k^L)^2}{\sqrt{x(1-x)(\mathcal{M} + 2m)}}, \\
\bar{u}_\uparrow(\gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m}) \cdot \epsilon_- v_\downarrow &= -\frac{\sqrt{2}(k_2^+ + m)k^L}{\sqrt{x(1-x)(\mathcal{M} + 2m)}}, \\
\bar{u}_\downarrow(\gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m}) \cdot \epsilon_- v_\uparrow &= \frac{\sqrt{2}(k_2^+ + m)k^L}{\sqrt{x(1-x)(\mathcal{M} + 2m)}}, \\
\bar{u}_\downarrow(\gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m}) \cdot \epsilon_- v_\downarrow &= -\frac{\sqrt{2}(m(\mathcal{M} + 2m) + k_1^2)}{\sqrt{x(1-x)(\mathcal{M} + 2m)}},
\end{aligned} \tag{21}$$

for the  $\rho^+$  meson with spin projection  $S_z = -1$ . After normalizations of the above matrix elements, we find

$$C_{S_z}^F(x, \vec{k}_\perp, \lambda_1, \lambda_2) = -\frac{1}{\sqrt{2}\mathcal{M}} \bar{u}(k_1^+, k_1^-, \vec{k}_\perp, \lambda_1) (\gamma - \frac{k_1 - k_2}{\mathcal{M} + 2m}) \cdot \epsilon_{S_z} v(k_2^+, k_2^-, -\vec{k}_\perp, \lambda_2). \tag{22}$$

Therefore, these two methods give exactly the same Fock expansion of the  $\rho^+$  meson and hence are equivalent to each other.

Similarly, we can find the Fock expansions of light-cone wave functions for other vector mesons by employing the same descriptions.

### 3. Pion-rho and rho-pion transition form factors

With the vanishing zero mode contribution in the pseudoscalar-vector transition process [8], the light-front Fock expansion provides a Lorentz-invariant representation of matrix elements of the electromagnetic current in terms of the overlap of light-front wave functions of hadrons. In section 2, the Fock state expansions of  $\rho$  and  $\pi$  light-cone wave functions are explicitly obtained. In this section, we discuss the pion-rho transition and rho-pion transition form factors.

The transition form factors of  $\gamma^* \pi \rightarrow \rho$  and  $\gamma^* \rho \rightarrow \pi$  are defined by

$$\begin{aligned}
\Gamma_{S_z,0}^\mu(\gamma^* \pi \rightarrow \rho) &= -ieF_{\gamma^* \pi \rightarrow \rho}(Q^2) \varepsilon^{\mu\nu\rho\sigma} \mathcal{P}_\nu \epsilon_{S_z,\rho} q_\sigma, \\
\Gamma_{S_z,0}^\mu(\gamma^* \rho \rightarrow \pi) &= -ieF_{\gamma^* \rho \rightarrow \pi}(Q^2) \varepsilon^{\mu\nu\rho\sigma} \mathcal{P}_\nu \epsilon_{S_z,\rho} q_\sigma,
\end{aligned} \tag{23}$$

in which  $\epsilon_{S_z}$  denotes the polarization vector of the rho meson and  $\mathcal{P} = \frac{1}{2}(P_\pi + P_\rho)$ . Here  $q = (P_\rho - P_\pi)$  is the four-momentum transfer of the virtual photon,  $q^2 = -Q^2 = q^+ q^- - \vec{q}_\perp^2 = -\vec{q}_\perp^2$ . Within the light-cone formalism, it is well known that the form factors can be uniquely determined by the matrix element of  $\Gamma^+$  component of the electromagnetic current,

$$\begin{aligned}
\Gamma_{S_z,0}^+(\gamma^* \pi \rightarrow \rho) &= \left\langle \psi_\rho(P_\rho^+, \vec{P}_{\perp\rho}, S_z) \right| J^+ \left| \psi_\pi(P_\pi^+, \vec{P}_{\perp\pi}, 0) \right\rangle \\
&\quad \times \delta^2(\vec{P}_{\perp\pi} + \vec{q}_\perp - \vec{P}_{\perp\rho}) \delta(P_\pi^+ + q^+ - P_\rho^+), \\
\Gamma_{0,S_z}^+(\gamma^* \rho \rightarrow \pi) &= \left\langle \psi_\pi(P_\pi^+, \vec{P}_{\perp\pi}, 0) \right| J^+ \left| \psi_\rho(P_\rho^+, \vec{P}_{\perp\rho}, S_z) \right\rangle \\
&\quad \times \delta^2(\vec{P}_{\perp\rho} + \vec{q}_\perp - \vec{P}_{\perp\pi}) \delta(P_\rho^+ + q^+ - P_\pi^+).
\end{aligned} \tag{24}$$

In the  $Q^2 \rightarrow 0$  limit, the  $\gamma^* \pi \rightarrow \rho$  transition form factor is the same as  $\gamma^* \rho \rightarrow \pi$  transition form factor, which is namely the magnetic moment of the pion-rho transition

$$\mu_{\pi\rho} = F_{\gamma^* \pi \rightarrow \rho}(0) = F_{\gamma^* \rho \rightarrow \pi}(0). \tag{25}$$

Another important static electromagnetic property of pion-rho and rho-pion transitions is the charge radius of the pion-rho transition, which can be obtained

via the  $Q^2 \rightarrow 0$  limit of pion-rho or rho-pion transition form factors as

$$\langle r_{\pi\rho}^2 \rangle = -6 \lim_{Q^2 \rightarrow 0} \frac{\partial F_{\gamma^* \pi \rightarrow \rho}(Q^2)}{\partial Q^2}. \quad (26)$$

In these calculations, the plus component of the local electromagnetic current reads

$$\frac{\bar{u}(k', \lambda')}{\sqrt{k'^+}} \gamma^+ \frac{\bar{u}(k, \lambda)}{\sqrt{k^+}} = 2 \delta_{\lambda', \lambda}, \quad (27)$$

and we choose the Drell-Yan assignment [6]:

$$\begin{aligned} q &= (q^+, q^-, \vec{q}_\perp) = \left(0, \frac{-q^2}{P^+}, \vec{q}_\perp\right), \\ P &= (P^+, P^-, \vec{P}_\perp) = \left(P^+, \frac{M^2}{P^+}, \vec{0}_\perp\right). \end{aligned} \quad (28)$$

In this  $q^+ = 0$  frame, the zero mode section (quark-antiquark pair creation graph) gives no contribution to transition form factors [8]. Thus, the matrix elements of space-like currents can be expressed as overlaps of light-cone wave functions with the same number of Fock constituents. In particular, for the transition form factors, we have

$$\begin{aligned} \frac{\Gamma_{S_z, 0}^+(\gamma^* \pi \rightarrow \rho)}{2P^+} &= \sum_a \int \frac{d^2 \vec{k}_\perp dx}{16\pi^3} \sum_{j, \lambda_1, \lambda_2} e_j \psi_{\rho, a}^{S_z*} \left(x_i, \vec{k}'_{\perp i}, \lambda_1, \lambda_2\right) \psi_{\pi, a} \left(x_i, \vec{k}_{\perp i}, \lambda_1, \lambda_2\right), \\ \frac{\Gamma_{0, S_z}^+(\gamma^* \rho \rightarrow \pi)}{2P^+} &= \sum_a \int \frac{d^2 \vec{k}_\perp dx}{16\pi^3} \sum_{j, \lambda_1, \lambda_2} e_j \psi_{\pi, a}^* \left(x_i, \vec{k}'_{\perp i}, \lambda_1, \lambda_2\right) \psi_{\rho, a}^{S_z} \left(x_i, \vec{k}_{\perp i}, \lambda_1, \lambda_2\right), \end{aligned} \quad (29)$$

where  $e_j$  is the charge of struck constituents and  $\psi_a \left(x_i, \vec{k}_{\perp i}, \lambda_i\right)$  is the light-cone Fock expansion wave function respectively. Here, for the final state light-cone wave function, the relative momentum coordinates are

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i) \vec{q}_\perp \quad (30)$$

for the struck quark and

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_\perp \quad (31)$$

for each spectator. It can be manifestly shown that the matrix element  $\Gamma_{0,0}^\mu$  is vanishing for any  $Q^2$  value and the matrix element  $\Gamma_{+1,0}^\mu$  is the same as  $\Gamma_{-1,0}^\mu$ . Therefore, evaluating  $\Gamma_{+1,0}^+$  by using the wave functions in equation (1), we obtain the explicit expressions for transition form factors.

For the  $\gamma^* \pi^+ \rightarrow \rho^+$  transition, we have calculated the light-cone valence quark wave function of the  $\rho^+$  and  $\pi^+$ . In the transition process, when  $u$  quark is the struck quark, the relative momentum coordinates of the final state are  $\vec{k}'_{\perp u} = \vec{k}_\perp + (1 - x) \vec{q}_\perp = \vec{k}'_\perp$  for the  $u$  quark and  $\vec{k}'_{\perp \bar{d}} = -\vec{k}_\perp - (1 - x) \vec{q}_\perp = -\vec{k}'_\perp$  for the spectator  $\bar{d}$ . The form factor is

$$F_{\gamma^* \pi \rightarrow \rho}^u(Q^2) = \frac{\Gamma_{+1,0}^+}{-ie(\vec{\epsilon}_\perp^* \times \vec{q}_\perp) \mathcal{P}^+} = e_u I(m, Q^2), \quad (32)$$



where

$$\begin{aligned}
I(m, Q^2) &= 2 \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \varphi_\rho^*(x, \vec{k}'_\perp) \varphi_\pi(x, \vec{k}_\perp) \\
&\quad \times \frac{m(\mathcal{M}' + 2m)(k'^L - k^L)q^R + k'^L q^R (k'^L k^R - k'^R k^L)}{Q^2(\mathcal{M}' + 2m) \sqrt{\vec{k}'_\perp{}^2 + m^2} \sqrt{\vec{k}_\perp{}^2 + m^2}} \\
&= 2 \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \varphi_\rho^*(x, \vec{k}'_\perp) \varphi_\pi(x, \vec{k}_\perp) \\
&\quad \times \frac{m(\mathcal{M}' + 2m)(1-x) + 2(1-x)\vec{k}_\perp^2 \sin^2 \alpha}{(\mathcal{M}' + 2m) \sqrt{\vec{k}'_\perp{}^2 + m^2} \sqrt{\vec{k}_\perp{}^2 + m^2}}, \tag{33}
\end{aligned}$$

and  $\mathcal{M}'^2 = \frac{\vec{k}'_\perp{}^2 + m^2}{x(1-x)}$ . Here,  $\alpha$  is the angle between  $\vec{k}_\perp$  and  $\vec{q}_\perp$ . Similarly, when  $\bar{d}$  quark is the struck quark, the relative momentum of the struck quark  $\bar{d}$  is  $\vec{k}'_{\perp \bar{d}} = -\vec{k}_\perp + x\vec{q}_\perp$  and that of the spectator  $u$  is  $\vec{k}_{\perp \bar{d}} = \vec{k}_\perp - x\vec{q}_\perp$ . If we let  $x \leftrightarrow (1-x)$  and  $\vec{q}_\perp \leftrightarrow -\vec{q}_\perp$  in the above formulae (This can be done because  $x$  is the integrate variable and form factor is only dependent on  $Q^2$ ), we directly see

$$F_{\gamma^* \pi \rightarrow \rho}^{\bar{d}}(Q^2) = -e_{\bar{d}} I(m, Q^2). \tag{34}$$

Therefore, the pion-rho transition form factor is

$$F_{\gamma^* \pi \rightarrow \rho}(Q^2) = F_{\gamma^* \pi \rightarrow \rho}^u(Q^2) + F_{\gamma^* \pi \rightarrow \rho}^{\bar{d}}(Q^2) = (e_u - e_{\bar{d}}) I(m, Q^2). \tag{35}$$

In a similar way, we can calculate the  $\gamma^* \pi^0 \rightarrow \rho^0$  transition. The SU(6) instant quark wave functions of the  $\rho^0$  meson are

$$\begin{aligned}
|\rho_\pm^0\rangle &= \frac{1}{\sqrt{2}} (u^{\uparrow, \downarrow} \bar{u}^{\uparrow, \downarrow} - d^{\uparrow, \downarrow} \bar{d}^{\uparrow, \downarrow}); \\
|\rho_0^0\rangle &= \frac{1}{2} (u^{\uparrow} \bar{u}^{\downarrow} + u^{\downarrow} \bar{u}^{\uparrow} - d^{\uparrow} \bar{d}^{\downarrow} - d^{\downarrow} \bar{d}^{\uparrow}), \tag{36}
\end{aligned}$$

and the wave function of the  $\pi^0$  meson is

$$|\pi^0\rangle = \frac{1}{2} (u^{\uparrow} \bar{u}^{\downarrow} - u^{\downarrow} \bar{u}^{\uparrow} - d^{\uparrow} \bar{d}^{\downarrow} + d^{\downarrow} \bar{d}^{\uparrow}). \tag{37}$$

After the Melosh rotation, the light-cone quark wave functions can be obtained. So the form factor is

$$\begin{aligned}
F_{\gamma^* \pi^0 \rightarrow \rho^0}(Q^2) &= \frac{1}{2} (F_{\gamma^* \pi \rightarrow \rho}^u(Q^2) + F_{\gamma^* \pi \rightarrow \rho}^{\bar{u}}(Q^2)) + \frac{1}{2} (F_{\gamma^* \pi \rightarrow \rho}^d(Q^2) + F_{\gamma^* \pi \rightarrow \rho}^{\bar{d}}(Q^2)) \\
&= \frac{1}{2} (e_u - e_{\bar{u}} + e_d - e_{\bar{d}}) I(m, Q^2) \\
&= F_{\gamma^* \pi^+ \rightarrow \rho^+}(Q^2). \tag{38}
\end{aligned}$$

We can find that equation (38) is the same as Eq. (32), because both the instant wave functions and the light-cone wave functions preserve the isospin symmetry. Thus at this level by assuming  $m_u = m_d = m$ , the  $\rho^\pm$  and  $\rho^0$  have identical  $\pi\gamma$  radiative decays. In this paper, we only consider the case of isospin symmetry as other model calculations [15, 13, 18], and will not distinguish the  $\rho^\pm$  transitions with  $\rho^0$  transitions in the following. In the Drell-Yan frame, the analytic continuation from space-like to time-like region only requires the change of  $\vec{q}_\perp$  to  $i\vec{q}_\perp$  in the form factor [27]. Choi and Ji [27] showed that the analytic continuation in the  $q^+ = 0$  frame exhibits the

same behavior as the direct analysis in  $q^+ \neq 0$  frame by counting the non-valence contribution.

Furthermore, the rho-pion transition form factor is

$$\begin{aligned}
F_{\gamma^* \rho \rightarrow \pi}(Q^2) &= \frac{\Gamma_{0,+1}^+}{-ie(\vec{\epsilon}_\perp \times \vec{q}_\perp) \mathcal{P}^+} \\
&= 2(e_u - e_d) \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \varphi_\pi^*(x, \vec{k}'_\perp) \varphi_\rho(x, \vec{k}_\perp) \\
&\quad \times \frac{m(\mathcal{M} + 2m)(k'^R - k^R)q^L + k^R q^L (k'^R k^L - k'^L k^R)}{Q^2(\mathcal{M} + 2m) \sqrt{\vec{k}_\perp^2 + m^2} \sqrt{\vec{k}'_\perp^2 + m^2}} \\
&= 2(e_u - e_d) \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \varphi_\pi^*(x, \vec{k}'_\perp) \varphi_\rho(x, \vec{k}_\perp) \\
&\quad \times \frac{m(\mathcal{M} + 2m)(1-x) + 2(1-x)\vec{k}_\perp^2 \sin^2 \alpha}{(\mathcal{M} + 2m) \sqrt{\vec{k}_\perp^2 + m^2} \sqrt{\vec{k}'_\perp^2 + m^2}}. \tag{39}
\end{aligned}$$

#### 4. Decay constants of the rho meson

Beside the transition form factors, we also compute the electromagnetic decay properties of the  $\rho$  meson by employing the light-cone wave functions of the  $\rho$  meson. Since the decay channels of the  $\rho$  meson are well measured, the decays constants can be used as the constraints to fix the parameters in numerical calculations.

- (i) Considering  $\rho \rightarrow \pi\gamma$  decay process, the decay width  $\Gamma(\rho \rightarrow \pi\gamma)$  has the following relationship with  $F_{\gamma^* \pi \rightarrow \rho}(0)$  and  $F_{\gamma^* \rho \rightarrow \pi}(0)$  [11]

$$\Gamma(\rho \rightarrow \pi\gamma) = \frac{\alpha(M_\rho^2 - M_\pi^2)^3}{24M_\rho^3} |F_{\gamma^* \pi \rightarrow \rho}(0)|^2 = \frac{\alpha(M_\rho^2 - M_\pi^2)^3}{24M_\rho^3} |F_{\gamma^* \rho \rightarrow \pi}(0)|^2. \tag{40}$$

So if we get the transition magnetic moment  $\mu_{\pi\rho}$ , we can calculate the decay width of the  $\rho \rightarrow \pi\gamma$  process.

- (ii) Another decay channel is the leptonic decay of the rho meson. The electromagnetic decay constant  $f_\rho$  is defined from  $\rho^0 \rightarrow e^+e^-$  [11] decay process. The width for this process is given, for leptons of zero mass, by

$$\Gamma(\rho^0 \rightarrow e^+e^-) = \frac{4\pi\alpha^2 f_\rho^2}{3M_\rho}. \tag{41}$$

The matrix element of the local vector current is defined as

$$\langle 0 | J_V^\mu | \psi^{S_z}(P^+, \vec{P}_\perp) \rangle = iM_\rho f_\rho \epsilon_{S_z}^\mu. \tag{42}$$

In light-cone formalism, the plus component of the matrix element can be written as

$$\begin{aligned}
\langle 0 | J_V^+ | P, h \rangle &= \frac{(e_u - e_d)}{\sqrt{2}} \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \varphi_\rho(x, \vec{k}_\perp) \\
&\quad \times \sum_{\lambda_1, \lambda_2} \bar{v}(k_2^+, k_2^-, -\vec{k}_\perp, \lambda_2) \gamma^+ u(k_1^+, k_1^-, \vec{k}_\perp, \lambda_1) \\
&\quad \bar{u}(k_1^+, k_1^-, \vec{k}_\perp, \lambda_1) \left( \gamma - \frac{k_1 - k_2}{M + 2m} \right) \cdot \epsilon(J^z) v(k_2^+, k_2^-, -\vec{k}_\perp, \lambda_2), \tag{43}
\end{aligned}$$

so we get

$$\frac{f_\rho}{2\sqrt{3}} = \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \frac{m(M+2m) + 2\vec{k}_\perp^2}{(M+2m)\sqrt{m^2 + \vec{k}_\perp^2}} \varphi_\rho(x, \vec{k}_\perp). \quad (44)$$

- (iii) From the electromagnetic decay, we can also relate the electromagnetic decay to the weak decay  $\rho^+ \rightarrow e^+ + \nu_e$  and  $\rho^+ \rightarrow \mu^+ + \nu_\mu$  by the following relation:

$$\Gamma(\rho^+ \rightarrow e^+ + \nu_e) = \frac{\alpha}{3} G_F^2 M_\rho^3 f_\rho^2 = \frac{G_F^2 M_\rho^4 \Gamma(\rho^0 \rightarrow e^+ e^-)}{4\pi\alpha}. \quad (45)$$

## 5. Numerical results and discussions

In the formula for the transition form factors  $F_{\gamma^* \pi \rightarrow \rho}(Q^2)$  and  $F_{\gamma^* \rho \rightarrow \pi}(Q^2)$ , there are five parameters: the  $\pi$  and  $\rho$  normalization constant  $A_\pi$  and  $A_\rho$ , the  $\pi$  harmonic scale of the momentum space wave function  $\beta_\pi$ , the  $\rho$  harmonic scale  $\beta_\rho$ , and the quark masses  $m$  (assuming  $m_u = m_d = m$  according to isospin symmetry). These parameters can be fixed by five constraints and hence we can proceed to calculate the transition form factors numerically, and predict other transition properties. We list these constraints as follows.

- (i) The weak decay constant  $f_\pi = (92.4 \pm 0.25)$  MeV [28] defined from  $\pi \rightarrow \mu\nu$  decay process by  $\langle 0 | \bar{u}\gamma^+(1 - \gamma_5)d | \pi \rangle = -\sqrt{2}f_\pi p^+$ , thus one obtains [20, 19]

$$\frac{f_\pi}{2\sqrt{3}} = \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \frac{m}{\sqrt{m^2 + \vec{k}_\perp^2}} \varphi_\pi(x, \vec{k}_\perp). \quad (46)$$

- (ii) The decay width  $\Gamma(\pi^0 \rightarrow \gamma\gamma)$  has the following relation with  $F_{\pi\gamma}(0)$  and  $F_{\gamma\pi}(0)$  [29]:

$$|F_{\gamma\gamma^* \rightarrow \pi}(0)|^2 = |F_{\gamma^* \pi \rightarrow \gamma}(0)|^2 = \frac{64\pi\Gamma(\pi^0 \rightarrow \gamma\gamma)}{(4\pi\alpha)^2 M_\pi^3}. \quad (47)$$

We use  $\Gamma(\pi^0 \rightarrow \gamma\gamma) = (7.74 \pm 0.54)$  eV [29], which leads to  $F_{\pi\gamma}(0) = (0.27 \pm 0.01)$  GeV<sup>-1</sup> in our calculation.

- (iii) The charge form factor of the pion has the following low-energy expansion:

$$F_{\pi^+}(Q^2) = 1 + \frac{1}{6} \langle r_{\pi^+}^2 \rangle Q^2 + \mathcal{O}(Q^4), \quad (48)$$

where  $\langle r_{\pi^+}^2 \rangle$  is the electromagnetic radius of the charged pion. Thus,

$$\langle r_{\pi^+}^2 \rangle = -6 \frac{\partial F_{\pi^+}(Q^2)}{\partial Q^2} \Big|_{Q^2=0}. \quad (49)$$

Experimentally, one finds [30]

$$\langle r_{\pi^+}^2 \rangle_{\text{exp}} = (0.439 \pm 0.03) \text{ fm}^2. \quad (50)$$

- (iv) The value of the transition form factor at  $Q^2 = 0$  (the so-called transition magnetic moment) have been experimentally determined from the radiative decay width of the  $\rho$  meson. The decay width of  $\rho^\pm \rightarrow \pi^\pm \gamma$  process is given by

$$\Gamma(\rho^\pm \rightarrow \pi^\pm \gamma) = \frac{\alpha(M_\rho^2 - M_\pi^2)^3}{24M_\rho^3} |F_{\gamma^* \pi^\pm \rightarrow \rho^\pm}(0)|^2 = \frac{\alpha(M_\rho^2 - M_\pi^2)^3}{24M_\rho^3} |\mu_{\pi\rho}|^2, \quad (51)$$

and the experimental value [31] is  $\Gamma(\rho^\pm \rightarrow \pi^\pm \gamma) = (68 \pm 7)$  keV. Hence, the value of the transition magnetic moment is

$$\mu_{\pi\rho} = F_{\gamma^* \pi \rightarrow \rho}(0) = (0.733 \pm 0.038) \text{ GeV}^{-1}. \quad (52)$$

**Table 1.** The three sets of parameters used in the calculations.

	Set I	Set II	Set III
$m_q$ (GeV)	0.200	0.220	0.220
$\beta_\pi$ (GeV)	0.410	0.420	0.420
$\beta_\rho$ (GeV)	0.410	0.420	0.360

**Table 2.** The static electromagnetic properties of pion and rho for the three sets of parameters.

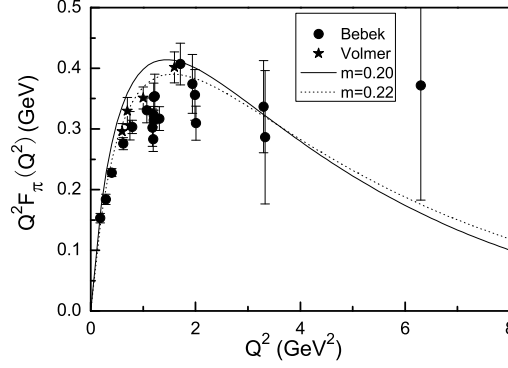
	Set I	Set II	Set III	Experimental
$f_\pi$ (MeV)	92.4	92.0	92.0	92.4
$\langle r_{\pi^+}^2 \rangle$ (fm <sup>2</sup> )	0.446	0.351	0.351	0.439
$F_{\pi \rightarrow \gamma\gamma}$ (GeV <sup>-1</sup> )	0.271	0.235	0.235	0.27
$f_\rho$ (MeV)	155.9	181.2	155.9	155.6
$F_{\rho \rightarrow \pi\gamma}$ (GeV <sup>-1</sup> )	0.731	0.736	0.769	0.733
$\langle r_{\pi\rho}^2 \rangle$ (fm <sup>2</sup> )	0.299	0.270	0.306	no Exp.

- (v) The experimental value [31] of the decay width for the  $\rho^0 \rightarrow e^+e^-$  process is  $\Gamma(\rho^0 \rightarrow e^+e^-) = (7.02 \pm 0.11)\text{keV}$ . From the Equation (41), the rho decay constant  $f_\rho$  can be determined from this process. The resulting rho decay constant is

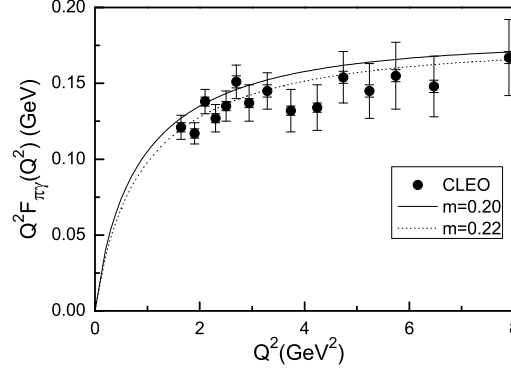
$$f_\rho = (155.6 \pm 19.5) \text{ MeV}. \quad (53)$$

From above five constraints, we can obtain the values of parameters. For the parameters of the pion meson, we still adopt the values of Ref. [19],  $m = 0.200$  GeV,  $\beta_\pi = 0.410$  GeV, and  $A_\pi = 47.5 \text{ MeV}^{-1}$ . For the other parameters about  $\rho$  meson, the values are  $\beta_\rho = 0.410$  GeV and  $A_\rho = 39 \text{ MeV}^{-1}$ . This is listed in the Set I of table 1. Conversely, using the above parameters, we can compute and predict the static properties and transition form factors. The results of static properties are shown in the Set I of table 2 and they are in very good agreement with above five experimental constraints. Furthermore, as shown in figure 1 and Figure 2, these fixed parameters provide predictions which are in good accordance with the experimental data for the pion charge form factors and pion-photon transition form factor, respectively. In addition, we can predict that the electromagnetic radii of form factors is  $\langle r_{\pi\rho}^2 \rangle = 0.299 \text{ fm}^2$ .

In above parametrization scheme, we adopt five experimental constraints to fix the parameters. In an alternative way, the normalizations of both  $\rho$  and  $\pi$  wave functions can be used as one kind of theoretical constraints. We use the value of the relativistic quark mass  $m = 0.22$  GeV which is frequently adopted. In set II of table 1, we simple use the same harmonic scales of the momentum space and in set III, the typical harmonic scale  $\beta = 0.360$  GeV is adopted to the  $\rho$  meson. In order to fix the harmonic scale of the pion meson we have to employ one experimental input:  $f_\pi$ . From the above theoretical constraints and the only experimental value of  $f_\pi$ , we can predict all other static properties which are listed in set II and set III of the table 2. Also, the pion charge form factors and pion-photon transition form factor from the predictions



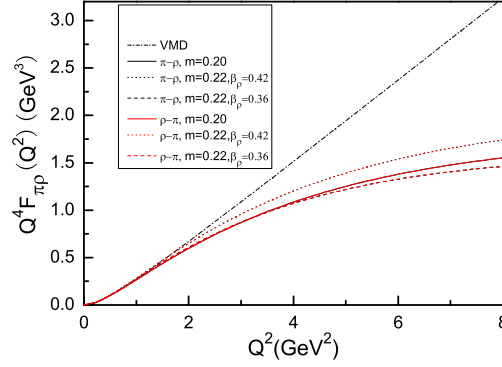
**Figure 1.** The charge form factor of the pion multiplied by  $Q^2$  in few GeV of the the momentum transfer. The experimental data are taken from [32] (full circles) and [33] (stars). The solid and dotted curves respectively correspond to the results of the first set and the other two sets of parameters.



**Figure 2.** The photon-pion transition form factor multiplied by  $Q^2$  compared with the experimental data. The data (full circles) are taken from [29]. The solid and dotted curves respectively correspond to the results of the first set and the other two sets of parameters.

by using these parameters are in a very good agreement with the experimental data, as shown in figure 1 and figure 2.

Figure 3 predicts the theoretical values of pion-rho and rho-pion space-like transition form factors in a range from low  $Q^2$  to  $Q^2 = 8 \text{ GeV}^2$ . Furthermore, we compare our results with the curve of the  $\rho$ -pole vector meson domination (VMD) model, where  $F_{\pi\rho}^{VMD}(Q^2) = F_{\pi\rho}(0)/(1 + Q^2/M_\rho^2)$ . Although  $F_{\rho\gamma^*\rightarrow\pi}(Q^2)$  and  $F_{\gamma^*\pi\rightarrow\rho}(Q^2)$  are physically different in equations (39) and (35), figure 3 indicates that they are numerically identical at very high precision, which implies that  $F_{\rho\gamma^*\rightarrow\pi}(Q^2)$  and  $F_{\gamma^*\pi\rightarrow\rho}(Q^2)$  have the same  $Q^2$  dependence. The identification of equations (35)



**Figure 3.** The space-like pion-rho and rho-pion transition for the  $Q^4 F_{\pi\rho}(Q^2)$  and  $Q^4 F_{\rho\pi}(Q^2)$ . The solid, dotted, and dashed curves respectively correspond to the pion-rho form factors by using the parameters of three sets of table 1, and the other curves correspond to the rho-pion form factors. Furthermore, the dash-dotted curve is the prediction of the  $\rho$ -pole vector meson domination (VMD) model.

and (39) can be proven by making the variable transformation:  $\vec{k}_\perp \rightarrow \vec{k}_\perp - (1-x)\vec{q}_\perp$  and then  $\vec{q}_\perp \rightarrow -\vec{q}_\perp$  for equation (39).

We can extend our discussion to the  $\gamma^* \pi^0 \rightarrow \omega^0$  transition. If we ignore the mixing of  $\omega$  and  $\phi$ , there are only the  $\gamma^* \pi \rightarrow \omega$  transition. The instant wave function of the  $\omega^0$  meson are

$$\begin{aligned} |\omega_\pm^0\rangle &= \frac{1}{\sqrt{2}}(u^\uparrow, \downarrow \bar{u}^\uparrow, \downarrow + d^\uparrow, \downarrow \bar{d}^\uparrow, \downarrow); \\ |\omega_0^0\rangle &= \frac{1}{2}(u^\uparrow \bar{u}^\downarrow + u^\downarrow \bar{u}^\uparrow + d^\uparrow \bar{d}^\downarrow + d^\downarrow \bar{d}^\uparrow). \end{aligned} \quad (54)$$

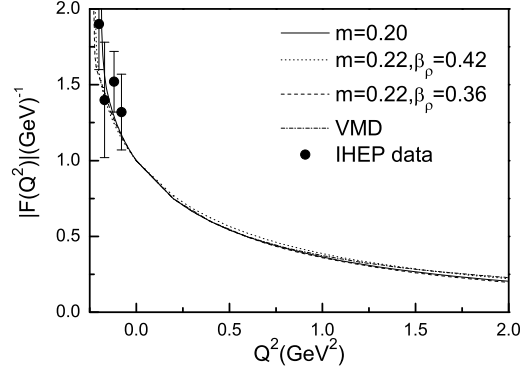
For simplicity, we use the same parameters as the  $\rho$  meson and hence the SU(6) symmetry has not been violated. The form factor can be written as

$$\begin{aligned} F_{\gamma^* \pi^0 \rightarrow \omega^0}(Q^2) &= \frac{1}{2}(F_{\gamma^* \pi \rightarrow \rho}^u(Q^2) + F_{\gamma^* \pi \rightarrow \rho}^{\bar{u}}(Q^2)) - \frac{1}{2}(F_{\gamma^* \pi \rightarrow \rho}^d(Q^2) + F_{\gamma^* \pi \rightarrow \rho}^{\bar{d}}(Q^2)) \\ &= \frac{1}{2}(e_u - e_{\bar{u}} - e_d + e_{\bar{d}})I(m, Q^2) \\ &= 3F_{\gamma^* \pi^+ \rightarrow \rho^+}(Q^2). \end{aligned} \quad (55)$$

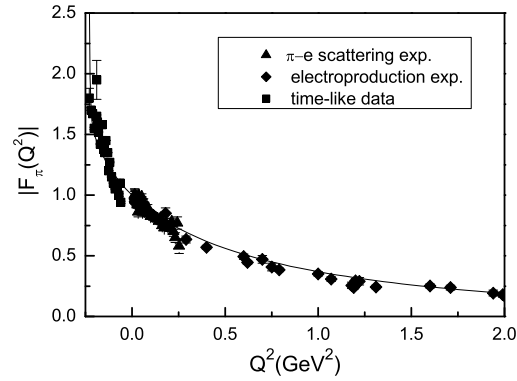
Again, this result is derived from the isospin symmetry. Therefore, if we adopt the first kind of parametrizations in table 1, we get the following predictions for the decay width of the  $\omega \rightarrow \pi\gamma$  transition and electromagnetic radius:

$$\begin{aligned} \Gamma(\omega \rightarrow \pi\gamma) &= \frac{\alpha(M_\omega^2 - M_\pi^2)^3}{24M_\omega^3} |F_{\omega \rightarrow \pi\gamma}(0)|^2 = (642 \pm 66) \text{ keV}, \\ \langle r_{\pi\omega}^2 \rangle &= 0.897 \text{ fm}^2. \end{aligned} \quad (56)$$

and the experimental value of the decay width is  $\Gamma^{\text{exp}}(\omega \rightarrow \pi\gamma) = (717 \pm 51) \text{ keV}$ . Furthermore, the transition form factor  $F_{\gamma^* \pi \rightarrow \omega}(Q^2)$  is three times larger than the form factor of the  $\gamma^* \pi \rightarrow \rho$  process.



**Figure 4.** Theoretical prediction for  $\gamma^*\pi \rightarrow \rho$  and  $\gamma^*\pi \rightarrow \omega$  space-like and time-like form factors, which have been normalized. Because of isospin symmetry, these two form factors are the same curve. The solid, dotted, and dashed curves respectively correspond to the form factors by using the parameters of three sets of table 1, and the dash-dotted curve is the prediction of the  $\rho$ -pole vector meson domination (VMD) model. The data in the time-like region are from [18, 34]



**Figure 5.** Pion space-like and time-like form factors as a function of the momentum transfer  $Q^2$ . The data in the spacelike region are from  $\pi$ - $e$  scattering experiments [35, 36, 37, 38] and electroproduction experiments [39, 39, 40, 41, 32, 33]. The data in the time-like region is from [42].

In figure 4, the space-like and time-like transition form factors are unified by analytical continuation technique. For simplicity, we normalize the  $\gamma^*\pi \rightarrow \rho$  and  $\gamma^*\pi \rightarrow \omega$  form factors in  $Q^2 = 0$  and so these two form factors become the same curve in the Figure 4. In the time-like region, beyond the threshold momentum transfer  $Q^2$  ( $Q^2 \simeq -0.2 \text{ GeV}^2$ ) the singularity for bound state production and the imagine part will appear. Choi and Ji [27] treated this case systematically for other processes. Now we only give the results below the particle production threshold. And in time-like region we can compare our numerical values with data [18, 34]. Besides, the same analytical continuation technique can also be used to the time-like pion form factor below threshold of particle production. In Figure 5, we only show the theoretical prediction by using the second set of the pion meson parameters in Table 1 and compare the results with the data of many experiments.

## 6. Conclusion

The light-cone formulism provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom, especially for the application to exclusive processes. To obtain the light-cone wave functions of the  $\rho$  meson, we employ both the light-cone quark model and rho-quark-antiquark vertex description which are equivalent with each other in the valence Fock-state expansion. Because of the vanishing zero mode by using this vertex, it is easy to derive the formulae of pion-rho transition form factors. Then we point out that in the isospin symmetry limit the form factor of the  $\gamma^*\pi^\pm \rightarrow \rho^\pm$  transition is equal to that of the  $\gamma^*\pi^0 \rightarrow \rho^0$  transition. We calculate the numerical results of space-like and time-like pion-rho and rho-pion transition form factors in terms of two groups of constraint parameters. The first one is that we adopt five experimental constraints to fix five parameters and conversely predict the static properties and transition form factors. At the same time, we explore the electro-weak decay properties of  $\rho$  and  $\pi$  mesons in light-cone formulism to connect the parameters with experimental constraints. Another parametrization is that we employ four theoretical constraints and only one experimental input  $f_\pi$  to reduce the dependence of experimental values and enhance the ability of prediction of this model. Finally, according to the isospin symmetry, we extend our calculation to the space-like and time-like  $\gamma^*\pi \rightarrow \omega$  processes and compare the theoretical result with time-like data.

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## References

- [1] Dirac P A M 1949 *Rev. Mod. Phys.* **21** 392  
Weinberg S 1966 *Phys. Rev.* **150** 1313
- [2] For a review and further references, see, Brodsky S J, Pauli H C and Pinsky S S 1998 *Phys. Rep.* **301** 299



- [3] de Melo J P B C, Frederico T, Pace E and Salme G 2006 *Phys. Rev. D* **73** 074013 and references therein
- [4] Lepage G P and Brodsky S J 1980 *Phys. Rev. D* **22** 2157  
Brodsky S J and Drell S D 1980 *Phys. Rev. D* **22** 2236
- [5] Brodsky S J, Hwang D S, Ma B Q and Schmidt I 2001 *Nucl. Phys.* **B593** 311
- [6] Drell S D and Yan T M 1970 *Phys. Rev. Lett.* **24** 181  
West G B 1970 *Phys. Rev. Lett.* **24** 1206
- [7] Bakker B L G, Choi H M and Ji C R 2002 *Phys. Rev. D* **65** 116001  
Choi H M and Ji C R 2004 *Phys. Rev. D* **70** 053015
- [8] Bakker B L G, Choi H M and Ji C R 2003 *Phys. Rev. D* **67** 113007  
Choi H M and Ji C R 2005 *Phys. Rev. D* **72** 013004
- [9] Jaus W 1999 *Phys. Rev. D* **60** 054026  
Jaus W 2003 *Phys. Rev. D* **67** 094010
- [10] Wigner E P 1949 *Ann. Math.* **49** 149  
Melosh H J 1974 *Phys. Rev. D* **9** 1095
- [11] Jaus W 1991 *Phys. Rev. D* **44** 2851
- [12] Brodsky S J, Huang T and Lepage G P 1983 *Particles and Fields-2 (Proceedings of the Banff Summer Institute, Banff, Alberta, 1981)* ed A Z Capri and A N Kamal (New York: Plenum)
- [13] Choi H M and Ji C R 1997 *Nucl. Phys.* **A618** 291  
Choi H M and Ji C R 1999 *Phys. Rev. D* **59** 074015
- [14] Ji C R, Chung P L and Cotanch S R 1992 *Phys. Rev. D* **45** 4214
- [15] Cardarelli F, Grach I L, Narodetsky I, Salme G and Simula S 1995 *Phys. Lett. B* **359** 1
- [16] Edwards R G 2005 *Nucl. Phys. B (Proc. Suppl.)* **140** 290
- [17] Khodjamirian A 1999 *Eur. Phys. J. C* **6** 477
- [18] Maris P and Tandy P C 2002 *Phys. Rev. C* **65** 045211
- [19] Xiao B W and Ma B Q 2003 *Phys. Rev. D* **68** 034020  
Xiao B W and Ma B Q 2005 *Phys. Rev. D* **71** 014034
- [20] Ma B Q 1993 *Z. Phys. A* **345** 321
- [21] Huang T, Ma B Q and Shen Q X 1994 *Phys. Rev. D* **49** 1490
- [22] Ma B Q 1991 *J. Phys. G: Nucl. Part. Phys.* **17** L53  
Ma B Q and Zhang Q R 1993 *Z. Phys. C* **58** 479
- [23] Ma B Q and Huang T 1995 *J. Phys. G: Nucl. Part. Phys.* **21** 765
- [24] Cao F G, Cao J, Huang T and Ma B Q 1997 *Phys. Rev. D* **55** 7107
- [25] Ma B Q 1996 *Phys. Lett. B* **375** 320  
Ma B Q and Schäfer A 1996 *Phys. Lett. B* **378** 307  
Ma B Q, Schmidt I and Soffer J 1998 *Phys. Lett. B* **441** 461  
Ma B Q, Schmidt I and Yang J J 2001 *Eur. Phys. J. A* **12** 353
- [26] Ma B Q and Schmidt I 1998 *Phys. Rev. D* **58** 096008
- [27] Choi H M and Ji C R 2001 *Nucl. Phys.* **A679** 735  
Choi H M and Ji C R 1998 *Phys. Rev. D* **59** 034001
- [28] Caso C *et al* 1998 *Eur. Phys. J. C* **3** 1
- [29] CLEO Collaboration, Gronberg J *et al* 1998 *Phys. Rev. D* **57** 33
- [30] Dally E B *et al* 1982 *Phys. Rev. Lett.* **48** 375
- [31] Particle Data Group, Yao W M 2006 *J. Phys. G: Nucl. Part. Phys.* **33** 1
- [32] Bebek C J *et al* 1978 *Phys. Rev. D* **17** 1693
- [33] Volmer J *et al* 2001 *Phys. Rev. Lett.* **86** 1713
- [34] Viktorov V A *et al* 1981 *Pisma Zh. Eksp. Teor. Fiz.* **33** 239
- [35] Adylov G T *et al* 1977 *Nucl. Phys.* **B128** 461
- [36] Dally E B *et al* 1981 *Phys. Rev. D* **24** 1718
- [37] Dally E B *et al* 1982 *Phys. Rev. Lett.* **48** 375
- [38] Amendolia S R *et al* 1986 *Nucl. Phys.* **B277** 168
- [39] Brown C N *et al* 1973 *Phys. Rev. D* **8** 92
- [40] Bebek C J *et al* 1974 *Phys. Rev. D* **9** 1229
- [41] Bebek C J *et al* 1976 *Phys. Rev. D* **13** 25
- [42] Baldini R, Dubnicka S, Gauzzi P, Pacetti S, Pasqualucci E and Srivastava Y 1999 *Eur. Phys. J. C* **11** 709  
Baldini R, Pasqualucci E, Dubnicka S, Gauzzi P, Pacetti S and Srivastava Y 2000 *Nucl. Phys.* **A666** 38